Midterm 2

Math 124, Fall 2026

Name: _____

uniqname: _____

UMID: _____

Room: ◯ 0460 CCCB ◯ 2866 East Hall ◯ 1866 East Hall

Instructions

- This exam consists of 8 problems across 14 pages, worth a total of 100 points.
- You have 120 minutes to complete this exam, unless you have extended-time accommodations through SSD.
- Write your uniqname in the top right corner of each page where a space is provided.
- For free response problems, **you must show all of your work**, and write your final answer in the small box when one is provided.
- For multiple choice problems, completely fill in bubbles and square boxes.
  ◯ A bubble means that you should only select one choice.
  ☐ A square box means that you should select all that apply.
- You may refer to **two double-sided** 4×6**-inch index cards**, handwritten directly on the cards. Other than that, you may not refer to any other resources or technology during the exam (no phones, watches, or calculators).

By signing below, I affirm that this work is my own and that I have followed the exam's resource and collaboration rules.

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uniqname: _____

Problem 1 (8 pts)

a) (4 pts) Let

$$C = \begin{bmatrix} 4 & -2 \\ -3 & 5 \\ 6 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} 2 & -4 & 3 \\ 5 & 1 & -2 \\ -3 & 6 & 4 \\ 7 & -2 & 5 \end{bmatrix}.$$

Give the shape of each expression (e.g. 5×3), or write "undefined."

CD		DC	
$C + C$		$D(3C)$	

b) (4 pts) Let a be a real constant, and let

$$A = \begin{bmatrix} a & 4 \\ 5 & a \\ -1 & -7 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 5 & -1 \end{bmatrix}.$$

Suppose the third column of AB is

$$\begin{bmatrix} 2 \\ m \\ 5 \end{bmatrix}.$$

i) (2 pts) Find the value of a. Show your work.

$a =$

ii) (2 pts) Find the value of m. Show your work.

$m =$

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Problem 2 (12 pts)

a) (8 pts) Suppose a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ satisfies

$$T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}, \quad T\left(\begin{bmatrix} -1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix}.$$

i) (1 pt) What is the shape of the matrix A where $T(\vec{x}) = A\vec{x}$?

Shape: [ ]

ii) (7 pts) Find an input vector $\vec{v} \in \mathbb{R}^2$ such that

$$T(\vec{v}) = \begin{bmatrix} 10 \\ -1 \\ -7 \end{bmatrix}.$$

$\vec{v} =$ [ ]

b) (4 pts) Which of the following transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$ are linear? **Select all that apply.**

- ☐ Multiply each vector by its length.
- ☐ Swap the horizontal and vertical components.
- ☐ Add 4 to the horizontal component and subtract 2 from the vertical component.
- ☐ Send every vector to the zero vector.

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Problem 3 (12 pts)

a) (4 pts) Let

$$A = \frac{1}{5}\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$

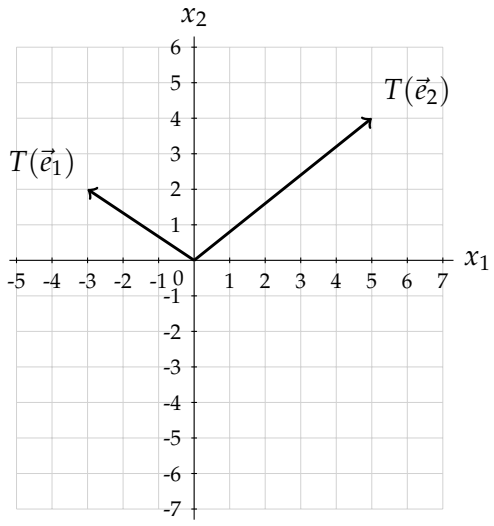
This matrix projects vectors orthogonally onto a line ℓ through the origin. Give a nonzero vector $\vec{u}$ such that $\ell = \text{span}(\vec{u})$.

$\vec{u} =$

b) (8 pts) The diagram shows $T(\vec{e}_1)$ and $T(\vec{e}_2)$ for a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, where the standard basis vectors are

$$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The transformation S first applies T, **then** rotates the resulting vector 90° **clockwise** about the origin. Find the 2×2 matrix B corresponding to S (**not** T).



$B =$

uniqname: _____

Problem 4 (12 pts)

a) (5 pts) Find A^{-1}, where

$$A = \begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix}.$$

$A^{-1} =$

b) (7 pts) Two invertible matrices C and D have inverses

$$C^{-1} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, \qquad D^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}.$$

A vector $\vec{x}$ is first multiplied by C, and the result is then multiplied by D. The final output is

$$\begin{bmatrix} 5 \\ 2 \end{bmatrix}.$$

Find $\vec{x}$.

$\vec{x} =$

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Problem 5 (10 pts)

Find matrices L and U such that $A = LU$, where

$$A = \begin{bmatrix} 5 & 2 & -4 \\ 15 & 3 & -5 \\ -10 & -1 & 7 \end{bmatrix}.$$

Here, L must be lower triangular with ones on its diagonal, and U must be upper triangular. No row swaps are necessary.

L

U

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Problem 6 (18 pts)

a) (10 pts) Suppose a 3×5 matrix A has the factorization $PA = LU$, where

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 4 & -8 & 3 & 2 & -5 \\ 0 & 0 & 5 & -10 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Use the given factorization to find all real values of c for which the following system has no solutions:

$$A\vec{x} = \begin{bmatrix} c \\ 4 \\ 7 \end{bmatrix}.$$

values of c: _____

uniqname: _____

b) (8 pts) Find all solutions of the following system. Give your answer in vector-parametric form.

$$\begin{bmatrix} 5 & -4 & 3 \\ 0 & 6 & -12 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -18 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

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Problem 7 (11 pts)

a) (8 pts) Let S be the plane $3x + 2y - z = 0$ in $\mathbb{R}^3$. Which of the following matrices A satisfy $\text{colsp}(A) = S$? **Select all that apply.**

☐ $A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \\ 4 & 3 \end{bmatrix}$

☐ $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 3 & 2 \\ 4 & 3 & 7 \end{bmatrix}$

☐ $A = \begin{bmatrix} 2 & 6 \\ -1 & -3 \\ 4 & 12 \end{bmatrix}$

☐ $A = \begin{bmatrix} 2 & -1 \\ -1 & 3 \\ 4 & 5 \end{bmatrix}$

b) (3 pts) Recall that a subspace contains the zero vector and is closed under addition and scalar multiplication. Consider the set

$$W = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : xy = 0 \right\}.$$

This set is **not** a subspace of $\mathbb{R}^2$, but it still satisfies some of these properties. For each property below, select **yes or no**.

i) (1 pt) Does W contain the zero vector?

◯ Yes ◯ No

ii) (1 pt) Is W closed under addition?

◯ Yes ◯ No

iii) (1 pt) Is W closed under scalar multiplication by any real number?

◯ Yes ◯ No

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Problem 8 (17 pts)

a) (6 pts) Let $\vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^5$ be linearly independent, and let $t \in \mathbb{R}$. Define

$$A = \begin{bmatrix} | & | \\ 2\vec{x} - \vec{y} & 6\vec{x} + t\vec{y} \\ | & | \end{bmatrix}, \qquad B = \begin{bmatrix} | & | & | \\ \vec{x} + \vec{y} & \vec{y} - 4\vec{z} & \vec{x} + t\vec{z} \\ | & | & | \end{bmatrix}.$$

For each part below, select the **one value of** t that satisfies the given condition.

i) (3 pts) $\text{rank}(A) = 1$

- ◯ $t = -4$
- ◯ $t = -3$
- ◯ $t = 0$
- ◯ $t = 1$
- ◯ $t = 3$
- ◯ $t = 4$

ii) (3 pts) $\text{rank}(B) = 2$

- ◯ $t = -4$
- ◯ $t = -3$
- ◯ $t = 0$
- ◯ $t = 1$
- ◯ $t = 3$
- ◯ $t = 4$

b) (2 pts) A matrix D has rank 8. The solutions of $D\vec{x} = \vec{0}$ have 5 free variables. How many columns does D have?

Number of columns: ☐

c) (6 pts) Suppose E is a 3×3 matrix such that

$$E\begin{bmatrix} 8 \\ -5 \\ 11 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 13 \end{bmatrix}, \qquad E\begin{bmatrix} -3 \\ 7 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

i) (2 pts) True or false: E is invertible.

◯ True ◯ False

ii) (4 pts) Give a solution of the following system **other than** $\begin{bmatrix} 8 \\ -5 \\ 11 \end{bmatrix}$:

$$E\vec{x} = \begin{bmatrix} 6 \\ -8 \\ 13 \end{bmatrix}.$$

$\vec{x} =$

d) (3 pts) A 6×4 matrix G has rank 4. Is $G\vec{x} = \vec{b}$ guaranteed to have a solution for every $\vec{b} \in \mathbb{R}^6$? Briefly explain.

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Feel free to use it for scratch work, but **do not tear it off**.

Congrats on finishing Midterm 2!

Feel free to draw us a picture about Math 124 in the box below (or use it for scratch work).