Practice Midterm 2

Math 124, Fall 2026

Name: _____

uniqname: _____

UMID: _____

Room: _____

Instructions

- This exam consists of **8** problems across **16** pages, worth a total of **100** points.
- You have 120 minutes to complete this exam, unless you have extended-time accommodations through SSD.
- Write your uniqname in the top right corner of each page where a space is provided.
- For free response problems, **you must show all of your work**, and write your final answer in the small box when one is provided.
- For multiple choice problems, completely fill in bubbles and square boxes.
  ◯ A bubble means that you should only select one choice.
  ☐ A square box means that you should select all that apply.
- You may refer to **two double-sided** 4×6**-inch index cards**, handwritten directly on the cards. Other than that, you may not refer to any other resources or technology during the exam (no phones, watches, or calculators).

By signing below, I affirm that this work is my own and that I have followed the exam's resource and collaboration rules.

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uniqname: _____

Problem 1 (8 pts)

Let c be a real constant, and let

$$A = \begin{bmatrix} 4 & -2 \\ -3 & 5 \\ 6 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 7 & 3 & -4 \\ 2 & c & 5 \end{bmatrix}.$$

a) (4 pts) Give the shape of each expression (e.g. 5×3), or write "undefined."

AB		BA	
$A + B$		$A(BA)$	

b) (4 pts) Suppose the second column of AB is

$$\begin{bmatrix} 20 \\ -29 \\ 14 \end{bmatrix}.$$

Find the value of c.

$c =$

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Problem 2 (10 pts)

a) (6 pts) Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation, and suppose

$$T\left(\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ -4 \end{bmatrix}, \qquad T\left(\begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 7 \end{bmatrix}.$$

i) (1 pt) What is the shape of the matrix corresponding to T, i.e., of the matrix A where $T(\vec{x}) = A\vec{x}$?

Shape: $\square$

ii) (5 pts) Find $T\left(\begin{bmatrix} -4 \\ 9 \\ 8 \end{bmatrix}\right)$ without finding the matrix of T.

$$T\left(\begin{bmatrix} -4 \\ 9 \\ 8 \end{bmatrix}\right) = \square$$

b) (4 pts) Which of the following transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$ are linear? **Select all that apply.**

- ☐ Rotate 30° clockwise about the origin, then add 1 to the horizontal component.
- ☐ Reflect across the line $y = -2x$, then multiply the vertical component by 4.
- ☐ Project orthogonally onto the line $y = 3x$.
- ☐ Replace the vertical component by its square, leaving the horizontal component unchanged.

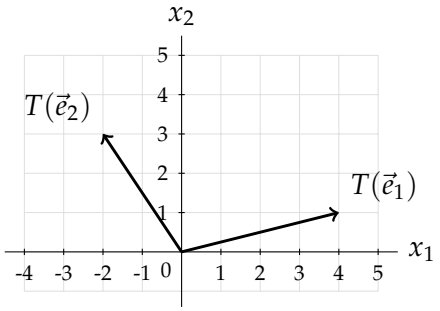
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Problem 3 (10 pts)

a) (6 pts) Find the 2×2 matrix P that projects each vector in $\mathbb{R}^2$ onto the line spanned by $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

$P =$

b) (4 pts) The diagram shows $T(\vec{e}_1)$ and $T(\vec{e}_2)$ for a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, where $\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find the matrix B corresponding to T.



$B =$

uniqname: _____

Problem 4 (13 pts)

a) (4 pts) An invertible 3×3 matrix A satisfies

$$A\begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad A\begin{bmatrix} -2 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \qquad A\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Find A^{-1} without first finding A. Explain how you found your answer.

$A^{-1} =$

b) (4 pts) Find B^{-1}, where

$$B = \begin{bmatrix} 5 & 4 \\ 6 & 8 \end{bmatrix}.$$

$B^{-1} =$

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c) (5 pts) An invertible matrix C has inverse

$$C^{-1} = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}.$$

Solve for $\vec{x}$:

$$C\left(4\vec{x} + \begin{bmatrix} 5 \\ -4 \end{bmatrix}\right) = \begin{bmatrix} 7 \\ 4 \end{bmatrix}.$$

$\vec{x} =$

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Problem 5 (16 pts)

a) (8 pts) Find a scalar k and matrices P, L, and U such that $PA = LU$, where

$$A = \begin{bmatrix} 0 & 3 & -4 \\ 4 & -2 & 6 \\ -4 & 8 & k \end{bmatrix}, \qquad U = \begin{bmatrix} 4 & a & b \\ 0 & 3 & c \\ 0 & 0 & 5 \end{bmatrix}.$$

Here, P is a permutation matrix, and L is lower triangular with ones on its diagonal. Write all entries of P, L, and U in the labeled boxes.

k P L U

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b) (8 pts) A different 3×3 matrix B has the factorization $PB = LU$, where

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \qquad L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & -2 & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} 3 & 4 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 4 \end{bmatrix}.$$

Use these factors to solve

$$B\vec{x} = \begin{bmatrix} 26 \\ 8 \\ 1 \end{bmatrix}.$$

For full credit, solve using two triangular systems; don't find the entries of B directly.

$\vec{x} =$

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Problem 6 (16 pts)

a) (8 pts) Let

$$A = \begin{bmatrix} 2 & -4 & 6 & 0 \\ -4 & 8 & -9 & 6 \\ 6 & -12 & 15 & -6 \end{bmatrix}.$$

Find all real numbers s for which the following system has infinitely many solutions:

$$A\vec{x} = \begin{bmatrix} s \\ 1 \\ 4 \end{bmatrix}.$$

If there are multiple such values of s, separate them with commas. You do not need to write the solution set.

$s =$ _____

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b) (8 pts) Find all solutions of the following system. Give your answer in vector-parametric form. Circle your final answer.

$$\begin{bmatrix} 2 & 0 & -4 & 6 & 0 \\ 0 & 3 & 6 & -3 & 0 \\ 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ -9 \\ 20 \\ 0 \end{bmatrix}.$$

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Problem 7 (11 pts)

a) (5 pts) Let

$$S = \left\{ \begin{bmatrix} 3a - 2b \\ 6a - 4b \\ 5a + b \end{bmatrix} : a, b \in \mathbb{R} \right\}.$$

Find a 3×3 matrix A such that $\text{colsp}(A) = S$.

$A =$

b) (6 pts) For each set below, explain why it is **not** a subspace of $\mathbb{R}^3$.

i) (2 pts)

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : z = 1 \right\}.$$

ii) (4 pts)

$$V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : y = 0,\ z = x^2 \right\}.$$

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Problem 8 (16 pts)

a) (8 pts) Let $\vec{p}, \vec{q}, \vec{r} \in \mathbb{R}^7$ be linearly independent. Find the rank of each matrix.

$$A = \begin{bmatrix} | & | & | & | \\ \vec{p}+\vec{q} & \vec{q}+\vec{r} & \vec{p}+\vec{r} & 4\vec{r} \\ | & | & | & | \end{bmatrix} \qquad \text{rank}(A) = \boxed{}$$

$$B = \begin{bmatrix} | & | \\ 2\vec{p}-\vec{q} & 8\vec{p}-4\vec{q} \\ | & | \end{bmatrix} \qquad \text{rank}(B) = \boxed{}$$

$$C = \begin{bmatrix} | & | & | \\ \vec{p}+2\vec{q} & 3\vec{q}-\vec{r} & 2\vec{p}+7\vec{q}-\vec{r} \\ | & | & | \end{bmatrix} \qquad \text{rank}(C) = \boxed{}$$

$$D = \begin{bmatrix} | & | & | & | \\ \vec{p}+\vec{q} & \vec{q}+\vec{r} & \vec{p}-\vec{r} & 2\vec{p}+\vec{q}-\vec{r} \\ | & | & | & | \end{bmatrix}. \qquad \text{rank}(D) = \boxed{}$$

b) (2 pts) A 6×8 matrix E has rank 3. How many free variables does $E\vec{x} = \vec{0}$ have?

Number of free variables: $\boxed{}$

c) (3 pts) A 4×7 matrix F has rank 4. Is every vector in $\mathbb{R}^4$ a possible output of F? Briefly explain.

d) (3 pts) A 6×4 matrix G has rank 4. Could $G\vec{x} = \vec{b}$ have exactly one solution for some $\vec{b} \in \mathbb{R}^6$? Briefly explain.

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Congrats on finishing Midterm 2!

Feel free to draw us a picture about Math 124 in the box below (or use it for scratch work).