# Lab 8: Inverses

Math 124, Fall 2026 at the University of Michigan

**due** by the end of class on Wednesday, October 21, 2026

**Name:** _____

**uniqname:** _____

Each lab worksheet will contain several activities, most of which will involve writing math on paper, and some of which will involve running code in a Jupyter Notebook. To receive credit for today's lab, you must show your lab TA your work on this worksheet.

While you must get checked off by your lab TA **individually**, we encourage you to form groups with 1-2 other students to complete the activities together.

**Review: Inverses.** For a square matrix $A$, an **inverse** is a matrix $A^{-1}$ satisfying

$$A^{-1}A = AA^{-1} = I.$$

Thus $A^{-1}(A\vec{x}) = \vec{x}$ for every input $\vec{x}$. A square matrix with an inverse is **invertible**; one without an inverse is **singular**.

### Activity 1: A quick check

Let $D = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$ and $E = \begin{bmatrix} -1/2 & 0 \\ 0 & 1/3 \end{bmatrix}$. Verify that $E = D^{-1}$ by computing both products in the definition above. Explain geometrically how $E$ undoes $D$.

## Activity 2: Undo the motion

Find the inverse of each matrix below **without** using the formula given in Activity 3. Instead, use geometry and/or your understanding of the given transformation. Verify your answers by multiplying.

**a)** The matrix $Q = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ rotates counterclockwise by $90°$. Describe the inverse transformation and write $Q^{-1}$.

**b)** The shear $S = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ sends $\begin{bmatrix} x \\ y \end{bmatrix}$ to $\begin{bmatrix} x+2y \\ y \end{bmatrix}$. Explain how to recover the original coordinates from the output, and use this to find $S^{-1}$.

**Activity 3: A formula for $2 \times 2$ inverses**

For a $2 \times 2$ matrix,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad \text{given } ad - bc \neq 0.$$

The number $ad - bc$ is called the **determinant** of $A$. If $ad - bc = 0$, then $A$ has no inverse.

**a)** Find the inverse of $A = \begin{bmatrix} 4 & -2 \\ 3 & 2 \end{bmatrix}$. Verify your result.

**b)** For which values of $t$ is $M = \begin{bmatrix} 4 & -2 \\ 3 & t \end{bmatrix}$ invertible? For the value of $t$ that makes $M$ singular, describe the span of its columns and explain why some output vectors cannot be reached.

**Activity 4: Recover the input**

Consider the equation

$$A\vec{x} = \vec{b},$$

where

$$A = \begin{bmatrix} 4 & -2 \\ 3 & 2 \end{bmatrix}, \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad \vec{b} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}.$$

You found the inverse of $A$ in Activity 3.

**a)** Write $A\vec{x} = \vec{b}$ as a system of two equations in the two unknowns $x_1$ and $x_2$. Then solve the system using $A^{-1}$ and verify your solution.

**b)** Suppose the first entry of $\vec{b}$ increases by $1$ while the second stays fixed. How do $x_1$ and $x_2$ change? Explain how your answer is encoded in a column of $A^{-1}$ and why the change does not depend on the original entries of $\vec{b}$.

**Activity 5: Undo a sequence**

An image editor first stretches horizontally by a factor of $2$, then applies a horizontal shear:

$$D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \qquad S = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Find the combined matrix $C$ and its inverse. Express $C^{-1}$ as a product involving $D^{-1}$ and $S^{-1}$, and explain the order in terms of undoing the two edits.

## Activity 6: Solving matrix equations

**a)** Suppose $A$ is a square matrix satisfying

$$A^4 - 3A^3 + A^2 + 3A + 5I = 0,$$

where $I$ and $0$ are the identity and zero matrices of the same shape as $A$. Find an expression for $A^{-1}$ in terms of powers of $A$, and justify why it is an inverse.

**b)** Suppose $A = BC$, where

$$A = \begin{bmatrix} 5 & -3 \\ 10 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}.$$

Find $C$.