Lab 9: LU Decomposition and Permutation Matrices

Math 124, Fall 2026 at the University of Michigan

due by the end of class on Wednesday, October 28, 2026

Name: _____

uniqname: _____

Each lab worksheet will contain several activities, most of which will involve writing math on paper, and some of which will involve running code in a Jupyter Notebook. Lab activities are meant to last an hour, and the second hour of lab is dedicated to starting the homework assignment. To receive credit for a lab, you must show your lab TA your work on both the lab worksheet and homework assignment.

While you must get checked off by your lab TA **individually**, we encourage you to form groups with 1-2 other students to complete the activities together.

How has the course content since fall break felt to you?

Circle one.

😭 😕 😐 🙂 😄

Do you have any concerns about the class that you would like to share?

Activity 1: Reordering a three by three matrix

Consider $A = \begin{bmatrix} 3 & -3 & 6 \\ 6 & -6 & 16 \\ -3 & 5 & -3 \end{bmatrix}$. Throughout, use ones on the diagonal of L.

a) Attempt to find an LU factorization without reordering the rows. Find the first row of U, then the first column of L, then the second row of U. At which step do you get stuck? Write the equation that makes it impossible to continue.

b) Choose two rows to swap so that the reordered matrix has an LU factorization. Write the reordered matrix, then restart the calculation and find its factors L and U.

c) Multiply your factors to check your answer. Does their product equal A or the reordered matrix?

Activity 2: A general two by two matrix

Let B be invertible, and write

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad L = \begin{bmatrix} 1 & 0 \\ \ell & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} p & q \\ 0 & r \end{bmatrix}.$$

a) Match entries in $B = LU$: first find the first row of U, then the first column of L, then the second row of U. At which step might you get stuck, and why?

b) For which invertible matrices does this procedure work without reordering the rows? Give formulas for L and U. Explain why both diagonal entries of U are nonzero and why the factors are uniquely determined.

c) If the procedure gets stuck, explain why swapping the rows allows you to continue. Use the invertibility of B to justify your answer, and find the factors for the reordered matrix.

Activity 3: Permutation matrices

A **permutation matrix** is a square matrix with exactly one 1 in each row and each column, and zeros everywhere else.

a) Find a 3×3 permutation matrix P such that left multiplication by P moves the first row of a matrix to the second row, the second row to the third row, and the third row to the first row.

b) Write down the inverses of the following matrices. Are their inverses also permutation matrices?

$$Q = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad R = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

c) Consider the three permutation matrices P_1, P_2, P_3 and the matrix B:

$$P_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & -1 & 4 \\ 0 & 3 & 5 \\ -2 & 6 & 1 \end{bmatrix}.$$

Find each product by observing how the entries are reordered, without explicitly carrying out matrix multiplication. Write each answer as a 3×3 matrix.

(i) P_1P_2 (ii) P_2B (iii) P_3B (iv) P_1P_2B (v) BP_1

d) Return to your factorization in Activity 1, where

$$A = \begin{bmatrix} 3 & -3 & 6 \\ 6 & -6 & 16 \\ -3 & 5 & -3 \end{bmatrix}.$$

Write your answer as $PA = LU$ for a permutation matrix P, and then as $A = P'LU$ for a permutation matrix P'. Write all matrices explicitly in both equations.