Lab 10: Column Space

Math 124, Fall 2026 at the University of Michigan

due by the end of class on Wednesday, November 4, 2026

Name: _____

uniqname: _____

Each lab worksheet will contain several activities, most of which will involve writing math on paper, and some of which will involve running code in a Jupyter Notebook. Lab activities are meant to last an hour, and the second hour of lab is dedicated to starting the homework assignment. To receive credit for a lab, you must show your lab TA your work on both the lab worksheet and homework assignment.

While you must get checked off by your lab TA **individually**, we encourage you to form groups with 1-2 other students to complete the activities together.

Activity 1: Column space basics

a) Fill in the blank: The **column space** of a matrix A, written $\text{colsp}(A)$, is the set of all [ ] [ ]s of A's [ ].

b) Let P be the plane

$$5x - y + 3z = 0.$$

Which of the following matrices have a column space equal to P? Circle all that apply.

$$A = \begin{bmatrix} 1 & 5 & 0 \\ 0 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 5 & 3 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 1 \\ 5 & 3 & 8 \\ 0 & 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & -1 \\ 5 & 10 & -5 \\ 0 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 \\ 5 & 1 \\ 0 & 0 \end{bmatrix}.$$

Activity 2: Two independent directions

Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

a) Fill in the blank: $\text{colsp}(A)$ is a $\square$ in $\mathbb{R}^{\square}$.

b) Find two vectors $\vec{u}$ and $\vec{v}$ such that

1. $\vec{u}$ and $\vec{v}$ are linearly independent.
2. $\text{span}(\vec{u}, \vec{v}) = \text{colsp}(A)$.

As we will see tomorrow, your $\vec{u}$ and $\vec{v}$ are a **basis** for $\text{colsp}(A)$.

c) Find two **different** 4×2 matrices whose column space is equal to $\text{colsp}(A)$.

Activity 3: Columns and invertibility

Definition: Linearly independent. The collection of vectors $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_d$ is **linearly independent** when none of the vectors in the collection can be written as a linear combination of the other vectors in the collection.

Fact: A square matrix is invertible if and only if its columns are linearly independent.

a) For each matrix, determine whether or not it is invertible.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

b) For which real values of t is D invertible?

$$D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & t \end{bmatrix}.$$

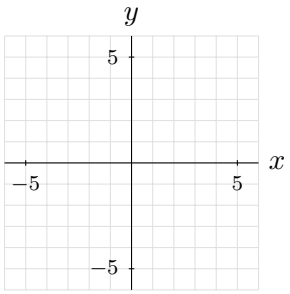
Activity 4: Subspaces

A **subspace** generalizes a line or plane through the origin: it is a set of vectors that contains $\vec{0}$ and is "flat" in some sense. Formally, a subset S of $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$ if:

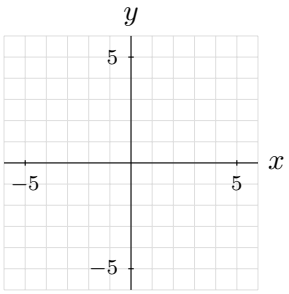
1. $\vec{0} \in S$.
2. If $\vec{u}, \vec{v} \in S$, then $\vec{u} + \vec{v} \in S$. We say S is **closed** under addition.
3. If $\vec{u} \in S$ and $c \in \mathbb{R}$, then $c\vec{u} \in S$. We say S is **closed** under scalar multiplication.

a) Sketch each set below. Each is a subset of $\mathbb{R}^2$, but only some are sub**spaces** of $\mathbb{R}^2$. Which and why?

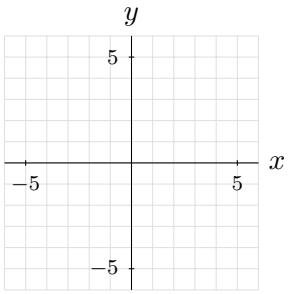
(i) Top right quadrant including its boundary
$x \geq 0,\ y \geq 0$



(ii) The line
$y = 2x$



(iii) The line
$y = 3x - 4$



b) Now consider the set

$$W = \left\{ \begin{bmatrix} a \\ b \\ a+b \end{bmatrix} : a, b \in \mathbb{R} \right\} \subseteq \mathbb{R}^3.$$

Find two linearly independent vectors $\vec{u}$ and $\vec{v}$ such that $\text{span}(\vec{u}, \vec{v}) = W$.