Lecture 13 Worksheet

Math 124, Fall 2026 at the University of Michigan

Some lectures will have an accompanying worksheet. We will do our best to include these problems directly in the notes as activities or worked examples.

Activity 1: Find the matrix

For each operation below, find the 2×2 matrix that performs it.

a) Stretch the x-coordinate by a factor of 3 and the y-coordinate by a factor of 2. Call this matrix D.

b) Reflect across the x-axis. Call this matrix F.

Activity 1: Find the matrix

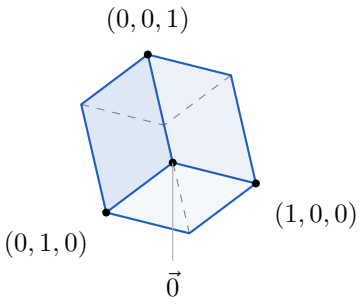
c) Project orthogonally onto the line spanned by $\begin{bmatrix}1\\2\end{bmatrix}$. Call this matrix P.

Reminder: For a nonzero vector $\vec{w}$, the projection of $\vec{v}$ onto its span is $\frac{\vec{v}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w}$.

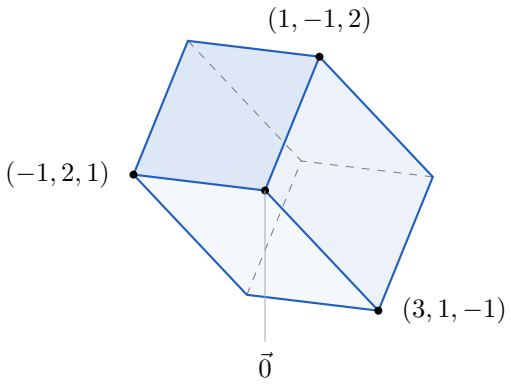
d) Rotate counterclockwise by 30° about the origin. Call this matrix R.

Activity 2: What matrix made this shape?

A linear transformation takes the cube on the left to the solid on the right.



Before



After

a) Find one 3×3 matrix that produces the solid on the right.

b) Could a different matrix produce the same solid? Try putting the same three columns in a different order. Write your new matrix and explain why the whole solid is the same.

Activity 3: Matrix multiplication

Golden rule: The number of columns in the first matrix must equal the number of rows in the second. The outer numbers give the shape of the answer.

a) Complete the table. Write "not defined" when a product cannot be computed.

Shape of A	Shape of B	Shape of AB	Shape of BA
5×7	7×9		
6×8	8×6		
10×10	10×10		

b) Compute the product.

$$\begin{bmatrix} 4 & -3 & 7 \\ -2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 5 & -4 \\ -2 & 8 \\ 3 & -6 \end{bmatrix}.$$

c) Compute the product.

$$\begin{bmatrix} 3 & -5 & 2 \\ -4 & 0 & 7 \\ 6 & 1 & -3 \end{bmatrix} \begin{bmatrix} 2 & 8 & -1 \\ 5 & -3 & 4 \\ -6 & 2 & 7 \end{bmatrix}.$$

Activity 3: Matrix multiplication

d) Let $H = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ stretch horizontally and $R = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ rotate counterclockwise by $90°$. Compute RH and HR. Are they equal? Would the order matter if $H = 2I$ instead? Explain without multiplying.

Activity 4: Can the final two steps be one matrix?

Use your matrices P and R from Activity 1. We want to **project onto the line spanned by** $\begin{bmatrix}1\\2\end{bmatrix}$ **first, then rotate counterclockwise by** $30°$.

a) Start with $\vec{v} = \begin{bmatrix}5\\0\end{bmatrix}$. Compute $P\vec{v}$, then $R(P\vec{v})$.

b) Can we find one matrix C that performs both steps for every input?
To find its columns, instead of multiplying matrices, follow the standard basis vectors through the two steps:

$$\vec{e}_1 \xrightarrow{P} P\vec{e}_1 \xrightarrow{R} R(P\vec{e}_1), \qquad \vec{e}_2 \xrightarrow{P} P\vec{e}_2 \xrightarrow{R} R(P\vec{e}_2).$$

Find these two final outputs and put them into columns to construct C.

c) Which matrix product, RP or PR, should equal C? Compute that product and compare it with your matrix from part (b). Check that $C\vec{v}$ agrees with part (a).

Activity 5: Distributivity: Keep the factors in order

Let

$$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}.$$

a) Compute $B + C$, then compute $A(B + C)$.

b) Compute AB and AC, then add your results. Compare $AB + AC$ with part (a). Write the distributive identity illustrated by these calculations.

c) Without substituting the numerical matrices, expand

$$(A + B)^2 = (A + B)(A + B).$$

Use distributivity and keep every product in its original order. Is the result always $A^2 + 2AB + B^2$? Explain.

Activity 6: What can the same output tell us?

In this activity, A and B are 2×2 matrices, and all vectors belong to $\mathbb{R}^2$. For each claim, explain why it must be true or give a counterexample. A counterexample must include specific matrices or vectors and the relevant products.

a) Suppose $A\vec{x} = B\vec{x}$ for one nonzero vector $\vec{x}$. Must $A = B$?

b) Now suppose $A\vec{x} = B\vec{x}$ for **every** vector $\vec{x} \in \mathbb{R}^2$. Must $A = B$? Explain what this tells us about the columns of A and B.

c) Suppose $A\vec{x} = A\vec{y}$. Must $\vec{x} = \vec{y}$? What could a transformation do that would cause different inputs to have the same output?