Lecture 16 Worksheet

Math 124, Fall 2026 at the University of Michigan

Some lectures will have an accompanying worksheet. We will do our best to include these problems directly in the notes as activities or worked examples.

Activity 1: A three by three factorization

Find an LU factorization of

$$A = \begin{bmatrix} 3 & 6 & -3 \\ -4 & -5 & 10 \\ 5 & 8 & -4 \end{bmatrix}$$

without reordering its rows. Use the shapes

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

a) Match entries of LU to entries of A. Find the first row of U, the first column of L, the second row of U, the second column of L, and finally u_{33}. Show your calculations and write your completed factors.

b) Multiply your completed matrices L and U to check that $LU = A$.

Activity 2: Using the factors to solve a system

Use the factors you found in Activity 1 to solve

$$\begin{bmatrix} 3 & 6 & -3 \\ -4 & -5 & 10 \\ 5 & 8 & -4 \end{bmatrix} \vec{x} = \begin{bmatrix} -3 \\ 7 \\ -2 \end{bmatrix}.$$

You do not need to find the factors again.

a) Set $\vec{y} = U\vec{x}$. Write the lower triangular system $L\vec{y} = \vec{b}$ and solve it by forward substitution. Write $\vec{y}$ as a column vector.

b) Write the upper triangular system $U\vec{x} = \vec{y}$ and solve it by back substitution. Write $\vec{x}$ as a column vector.

c) Check your solution by multiplying the original matrix A by your vector $\vec{x}$.

Activity 3: A different coefficient matrix

Consider the system

$$\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}.$$

a) Try to find an LU factorization of the coefficient matrix using

$$L = \begin{bmatrix} 1 & 0 \\ \ell & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} p & q \\ 0 & r \end{bmatrix}.$$

Which entry of LU makes this impossible?

b) Reorder the equations so that the new coefficient matrix has an LU factorization. Write the reordered matrix equation, including its right-hand side, and find L and U.

c) Use these factors to solve the reordered system. Check your answer in the original system. Does the failure to find factors in part (a) mean the original system has no solution?

Activity 4: A three by three matrix that needs a row swap

Consider

$$B = \begin{bmatrix} 2 & 1 & -1 \\ 4 & 2 & 1 \\ -2 & 2 & 3 \end{bmatrix}.$$

Throughout, use ones on the diagonal of L.

a) Attempt to find an LU factorization without reordering the rows. Follow the same order as in Activity 1. At which step do you get stuck? Write the equation that makes it impossible to continue.

b) Choose two rows to swap so that the reordered matrix has an LU factorization. Write the reordered matrix, then restart the calculation and find its factors L and U.

c) Multiply your factors to check your answer. Does their product equal B or the reordered matrix?

Activity 5: A general two by two matrix

Let A be invertible, and write

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad L = \begin{bmatrix} 1 & 0 \\ \ell & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} p & q \\ 0 & r \end{bmatrix}.$$

a) Match entries in $A = LU$: first find the first row of U, then the first column of L, then the second row of U. At which step might you get stuck, and why?

b) For which invertible matrices does this procedure work without reordering the rows? Give formulas for L and U. Explain why both diagonal entries of U are nonzero and why the factors are uniquely determined.

c) If the procedure gets stuck, explain why swapping the rows allows you to continue. Use the invertibility of A to justify your answer, and find the factors for the reordered matrix.