# Lecture 17 Worksheet

Math 124, Fall 2026 at the University of Michigan

Some lectures will have an accompanying worksheet. We will do our best to include these problems directly in the notes as activities or worked examples.

## Activity 1: Recognizing row echelon form

For each matrix, decide whether it is in row echelon form. If it is, circle its pivots. If it is not, explain which requirement fails.

**a)**

$$\begin{bmatrix} 3 & -1 & 2 \\ 0 & 2 & 5 \\ 0 & 0 & -4 \end{bmatrix}$$

**b)**

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

**c)**

$$\begin{bmatrix} 0 & 2 & -1 & 4 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

**d)**

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -2 \end{bmatrix}$$

**e)**

$$\begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$$

**f)**

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & 0 & -1 \end{bmatrix}$$

**g)**

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

**h)**

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

## Activity 2: Finding all solutions

Find **all solutions** of each system. First check whether the system is consistent. If it is, keep the variables in pivot columns on the left and move the other variables to the right. Identify any free variables and solve for the remaining variables.

Write your answer using column vectors. If there are infinitely many solutions, give a parametric description. If there is no solution, explain why.

**a)**

$$\begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 9 \\ 0 \end{bmatrix}.$$

**b)**

$$\begin{bmatrix} 2 & -2 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 0 \end{bmatrix}.$$

**c)**

$$\begin{bmatrix} 3 & 1 & -2 \\ 0 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 2 \end{bmatrix}.$$

**d)**

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}.$$