Lecture 21 Worksheet

Math 124, Fall 2026 at the University of Michigan

Some lectures will have an accompanying worksheet. We will do our best to include these problems directly in the notes as activities or worked examples.

Activity 1: Transposes and orthonormal columns

a) Let $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$. Compute $(AB)^T$ and $B^T A^T$, and check that they agree. Does $A^T B^T$ give the same matrix?

b) Let $Q = \begin{bmatrix} 12/13 & 5/13 \\ 5/13 & -12/13 \end{bmatrix}$. Show that Q is orthogonal, write down Q^{-1}, and find $Q \begin{bmatrix} 2 \\ -3 \end{bmatrix}$. Check that the length is unchanged.

c) Let $C = \begin{bmatrix} 2/7 & 3/7 \\ 3/7 & -6/7 \\ 6/7 & 2/7 \end{bmatrix}$. Compute C^TC and CC^T. Are the columns orthonormal? Is C an orthogonal matrix?

Activity 2: Combining rotations and reflections

First, make a guess. In $\mathbb{R}^2$, what kind of transformation do you get by doing each of the following? Predict a rotation or a reflection before trying any examples.

A rotation, then a reflection: _____

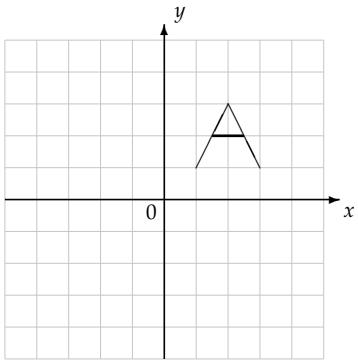
A reflection, then a rotation: _____

Two reflections: _____

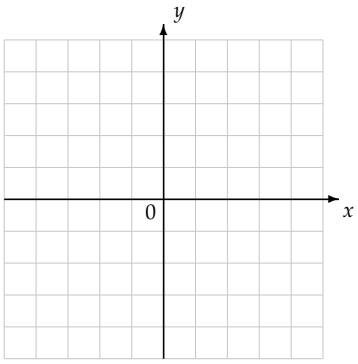
Now test your guesses with the letter A. For each subpart, draw the image after the first transformation in the middle grid, then its image after the second transformation in the right-hand grid. Use solid lines throughout. Each square represents one unit. All rotations are about the origin.

a) Reflect across the x-axis, then rotate 90° counterclockwise.

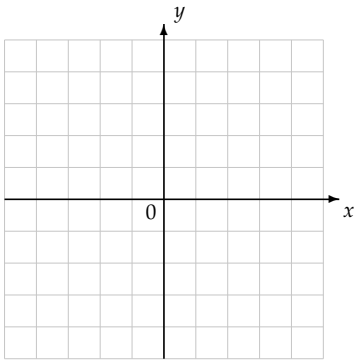
Original



After first step

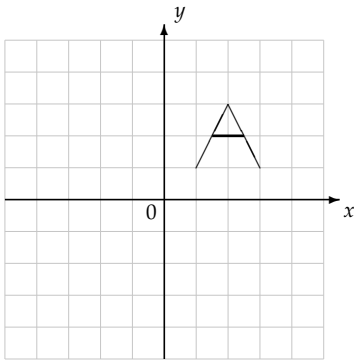


After second step

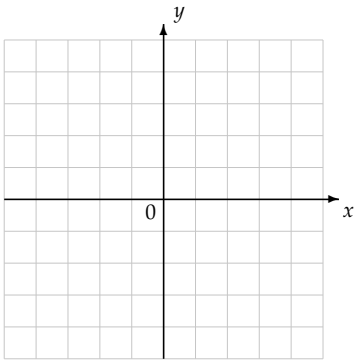


b) Rotate 90° counterclockwise, then reflect across the x-axis.

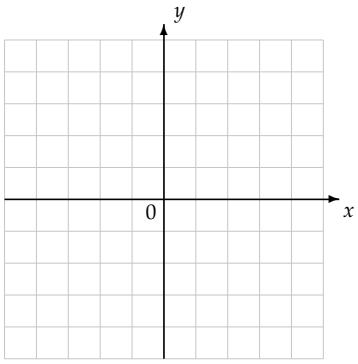
Original



After first step



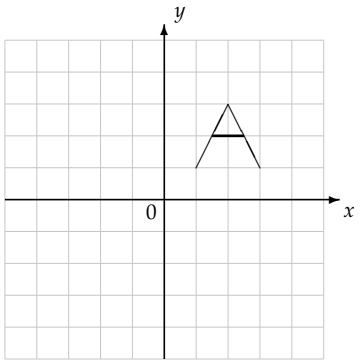
After second step



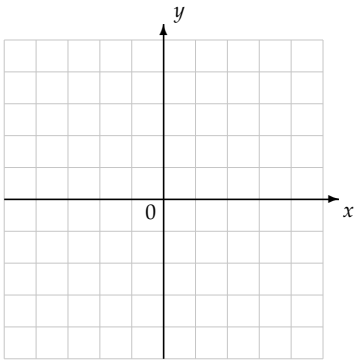
Compare (a) and (b). Do the pictures support your guesses? Does the order matter? Identify the single transformation in each case, including its angle or reflecting line.

c) Reflect across the x-axis, then reflect across the line $y = x$.

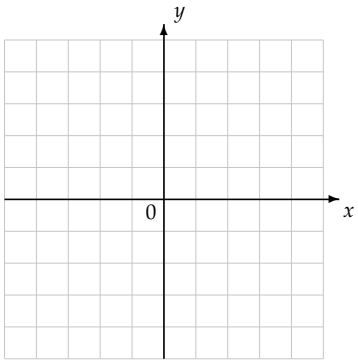
Original



After first step

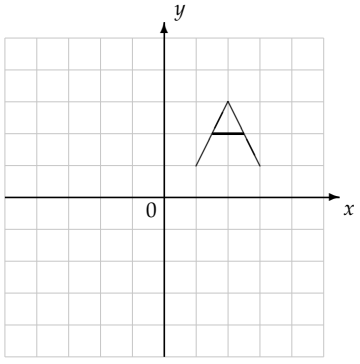


After second step

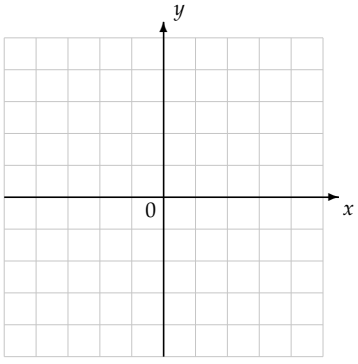


d) Reflect across the line $y = x$, then reflect across the x-axis.

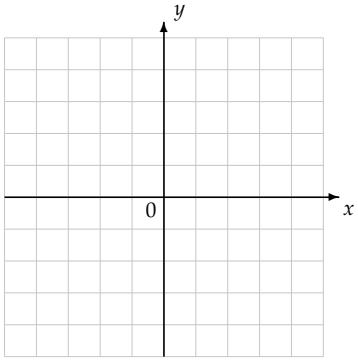
Original



After first step



After second step



Compare (c) and (d). Do the pictures support your guess? Identify the single transformation in each case. What changes when you reverse the order?

Activity 3: What might happen in $\mathbb{R}^3$?

In $\mathbb{R}^2$, every orthogonal transformation is a rotation or a reflection. What kinds of orthogonal transformations might there be in $\mathbb{R}^3$? Discuss the possibilities and make a guess about how you might describe them all.